THE EUCLIDEAN ALGORITHM FOR FINDING $\gcd(a,b)$

when $a > b$ and both are > 0.

STEP

① DIVIDE

$$\begin{array}{r} q_1 \\ b\,\big)\overline{\,a\,} \\ \Downarrow \\ \hline r_1 \end{array}$$

a = the larger of the two #'s

If $r_1 = 0$, $b \mid a$ and $\gcd(a,b) = b$

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② Divide

$$\begin{array}{r} q_2 \\ r_1\,\big)\overline{\,b\,} \\ \Downarrow \\ \hline r_2 \end{array}$$

$0 < r_2 < r_1$

$\vdots$

Ⓚ DIVIDE

$$\begin{array}{r} q_k \\ r_{k-1}\,\big)\overline{\,r_{k-2}\,} \\ \Downarrow \\ \hline r_k \end{array}$$

$0 < r_k < r_{k-2}$

(k+1) DIVIDE

$$\begin{array}{r} q_{k+1} \\ r_k\,\big)\overline{\,r_{k-1}\,} \\ \Downarrow \\ \hline 0 \end{array}$$

0 = when Remainder $= 0$, STOP!

$\gcd(a,b) = r_k$ = The LAST NON-ZERO REMAINDER

EXAMPLE APPLICATIONS OF THE EUCLIDEAN ALGORITHM:

FIND gcd(63, 72)

$$\begin{array}{r} 1 \\ 63\overline{)72} \\ -63 \\ \hline \underline{9} \end{array} \dashrightarrow \begin{array}{r} 7 \\ 9\overline{)63} \\ -63 \\ \hline 0 \end{array}$$

STOP!

gcd(63,72) = 9

FIND gcd(3258, 642)

$$\begin{array}{r} 5 \\ 642\overline{)3258} \\ -3210 \\ \hline 48 \end{array} \dashrightarrow \begin{array}{r} 13 \\ 48\overline{)642} \\ 48 \\ \hline 162 \\ 144 \\ \hline 18 \end{array} \dashrightarrow \begin{array}{r} 2 \\ 18\overline{)48} \\ -36 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 1 \\ 12\overline{)18} \\ -12 \\ \hline \underline{6} \end{array} \qquad \begin{array}{r} 2 \\ 6\overline{)12} \\ -12 \\ \hline 0 \end{array}$$

STOP!

gcd(642,3258) = 6

Be sure to read Example 8.5.2 beginning at bottom of page 390 of the EPP BRIEF EDITION TEXTBOOK.